Transport coefficients of $O(N)$ scalar field theories close to the critical point*

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In recent decades transport coefficients of quantum chromodynamics (QCD) have attracted much interest in the context of Relativistic Heavy Ion Collider (RHIC) experiment, which are aimed at creating and studying the quark-gluon plasma. One of the interesting experimental findings, the large elliptic flow v_2 observed in high energy non-central collisions, implies that the spatial anisotropy of the initial state is, during the expansion, very efficiently converted to a transverse momentum anisotropy of the observed hadrons. These experimental results are well described by ideal hydrodynamics with vanishing viscosity. Thus, the large elliptic flow observed in such collisions implies that the fireball created behaves as an almost perfect fluid.

Although the transport coefficients in viscous hydrodynamics are phenomenological parameters, they can in principle be computed from a microscopic starting point. Since the shear viscosity, one of transport coefficients, has a direct influence on the elliptic flow, the experimental results have triggered numerous theoretical efforts to unravel its behavior as a function of thermodynamic variables. By and large, these are performed in the framework of kinetic theory, e.g. using the Boltzmann equation, applied to effective theories of QCD and to perturbative QCD. Furthermore, some results on the temperature dependence of the transport coefficients have been obtained in lattice simulations.

The experimental results have motivated recent work on a field theoretical approach to evaluate transport coefficients. The $O(N)$ scalar field theory offers a testing ground for developing computational methods before facing the complications of a full QCD calculation. In the present work we discuss the critical behavior of the shear viscosity and other transport coefficients in the $O(N)$ scalar field theory. As demonstrated by Wilson using the renormalization group, there is a second-order phase transition in the $O(N)$ scalar field theory with a finite ultraviolet cut-off Λ. Kinetic approaches employed for computing the transport coefficients rely heavily on Boltzmann-like approximations, which take only the single particle distribution into account and neglect higher order correlations. Although these correlations, may be unimportant far from the critical point, they play a key role in the critical region.

In our study of the critical transport properties, we employ the dynamical renormalization group (DRG) combined with the epsilon expansion. This method was tested for model H developed by Hohenberg and Halperin [1]. Within this approach we examine the scale evolution of a stochastic equation of motion, which describes the critical dynamics of the slow modes. These include fluctuations of the order parameter and of conserved quantities. These are the relevant variables when addressing the long-wave length behavior of the system near the critical point.

In analogy to the static case, the flow equations for transport coefficients derived from the DRG admit non-trivial fixed points, from which the *dynamical critical exponent*, z, and the dynamical scaling relations can be deduced. The dynamical critical exponent, z, defines the characteristic frequency of the most relevant slow mode $\omega \sim k^z$. Using the DRG approach, one can also derive scaling relations for the transport coefficients, and deduce the singular behavior of the transport coefficients. Based on the universal behavior, i.e. on the dynamical critical exponents and scaling laws, one identifies each systems with a dynamical universality class. In contrast to the static case, the dynamical universality class is governed not only by the dimensionality, locality, and the symmetries of the system under consideration, but, in addition, by the properties of the relevant slow modes.

In this work [2], we determine the dynamical universality class of the $O(N)$ scalar field theory and show how the dynamical universality class depends on the number of components, N, and on the dimensionality, d. As demonstrated in Ref. [2], the dynamical universality class of the single component theory reduces to that of model C. The dynamical critical exponent is given by $z = 2+\alpha/\nu$, where α and ν are the static critical exponents of $O(N)$ model. On the other hand, for the multicomponent theory ($N > 1$), the critical dynamics is dominated by $O(N)$ charge fluctuations. This drives the critical exponent down to the value $z = d/2$ and the theory belongs to the dynamic universality class of model G. In both cases, $N = 1$ and $N > 1$, the shear viscosity remains finite at the critical point, while the bulk viscosity diverges for $N = 1$ with the dominant singular contribution proportional to $\zeta \sim \xi^{z-\alpha/\nu} \sim \xi^2$, and remains finite for $N > 1$. In QCD, the $O(4)$ chiral symmetry in the light quark sector is broken by the non-zero u and d quark masses. For high temperatures and small values of the chemical potential, the second-order phase transition is replaced by a crossover. Our results imply that the singular part of the shear and bulk viscosity remain finite also at the QCD phase transition.

References

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